A principled derivation of OT and HG within constraint-based phonology

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QUESTION

We consider:

- a set Gen of phonological mappings
- a set of n relevant constraints
- an order $\prec$ among n-dimensional vectors: $\boldsymbol{\alpha} \prec \boldsymbol{\beta}$ iff $\boldsymbol{\alpha}$ smaller than $\boldsymbol{\beta}$

The **constraint-based grammar** $G_{\prec}$ realizes an underlying form x as a surface candidate y $\in Gen(\mathsf{x})$ that is optimal because no other candidate z $\in Gen(\mathsf{x})$ has a smaller vector of constraint violations wrt $\prec$

There are scores of orders $\prec$ among n-dimensional vectors: why do phonologists only use:

- OT's **lexicographic** order and
- HG's **linear** order?

INTUITION

$$\frac{G(/ad/) = [ad] \qquad G(/tada/) = [tada]}{G(/adtada/) = [adtada]}$$

This syllogism is wrong: G might ban clusters and one such marked structure *dt* is **created** by concatenating *ad* and *tada* into *adtada*

$$\frac{G(/adt/) = [ad] \qquad G(/ada/) = [ada]}{G(/adtada/) = [adada]}$$

This syllogism is wrong: G might ban complex codas and one such marked structure is **dissolved** when *adt* and *ada* are syllabified together as *adtada*

$$\frac{G(/adta/) = [adta] \qquad G(/da/) = [da]}{G(/adtada/) = [adtada]}$$

This syllogism seeems correct: the concatenation of *adta* and *da* into *adtada* is **innocuous**: it doesn't create nor dissolve relevant marked structures

FORMALIZING THE INTUITION

A surface concatenation $\boldsymbol{y}' \cdot \boldsymbol{y}''$ is **innocuous** (wrt M) provided it neither creates nor dissolves markedness violations: $M(\boldsymbol{y}' \cdot \boldsymbol{y}'') = M(\boldsymbol{y}') + M(\boldsymbol{y}'')$ for every markedness constraint M in M

An underlying concatenation $\boldsymbol{x}' \cdot \boldsymbol{x}''$ is **innocuous** (wrt Gen and M) provided the concatenation $\boldsymbol{y}' \cdot \boldsymbol{y}''$ of any two of their surface candidates $\boldsymbol{y}'$ and $\boldsymbol{y}''$ from $Gen(\boldsymbol{x}')$ and $Gen(\boldsymbol{x}'')$ is innocuous

The intuition above can now be formalized as the following **axiom**: whenever an underlying concatenation $\boldsymbol{x}' \cdot \boldsymbol{x}''$ is innocuous, $G(\boldsymbol{x}' \cdot \boldsymbol{x}'') = G(\boldsymbol{x}') \cdot G(\boldsymbol{x}'')$

Example: the underlying strings /adta/ and /da/ with candidates obtained by changing obstruent voicing are innocuous relative to NoCompCoda, AgreeVoice, and NoVoice, as verified here. The axiom thus requires grammars to satisfy $G(/adtada/) = G(/adta/) \cdot G(/da/)$

(/adtada/, [attata]) $\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\0\end{bmatrix}$ (/adtada/, [adtata]) $\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\1\\1\end{bmatrix} + \begin{bmatrix}0\\0\\0\end{bmatrix}$ (/adtada/, [atdata]) $\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\1\\1\end{bmatrix} + \begin{bmatrix}0\\0\\0\end{bmatrix}$ (/adtada/, [addata]) $\begin{bmatrix}0\\0\\2\end{bmatrix} = \begin{bmatrix}0\\0\\2\end{bmatrix} + \begin{bmatrix}0\\0\\0\end{bmatrix}$ | (/da/, [ta]) $\begin{bmatrix}0\\0\\1\end{bmatrix}$

(/adtada/, [attada]) $\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}$ (/adtada/, [adtada]) $\begin{bmatrix}0\\1\\2\end{bmatrix} = \begin{bmatrix}0\\1\\1\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}$ (/adtada/, [atdada]) $\begin{bmatrix}0\\1\\2\end{bmatrix} = \begin{bmatrix}0\\1\\1\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}$ (/adtada/, [addada]) $\begin{bmatrix}0\\0\\3\end{bmatrix} = \begin{bmatrix}0\\0\\2\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}$ | (/da/, [da]) $\begin{bmatrix}0\\0\\0\end{bmatrix}$

NoCompCoda / AgreeVoice / NoVoice: (/adta/, [atta]) $\begin{bmatrix}0\\0\\0\end{bmatrix}$ (/adta/, [adta]) $\begin{bmatrix}0\\1\\1\end{bmatrix}$ (/adta/, [atda]) $\begin{bmatrix}0\\1\\1\end{bmatrix}$ (/adta/, [adda]) $\begin{bmatrix}0\\0\\2\end{bmatrix}$

MAIN RESULT

An order $\prec$ among n-dimensional vectors yields a constraint-based grammar $G_{\prec}$ that satisfies the axiom if and only if there exist some d **weight** vectors $\mathbf{w}^{(1)} = (w_1^{(1)}, \ldots, w_n^{(1)}) \ldots \mathbf{w}^{(d)} = (w_1^{(d)}, \ldots, w_n^{(d)})$ such that two arbitrary n-dimensional vectors $\boldsymbol{\alpha} = (\alpha_1 \ldots \alpha_n)$ and $\boldsymbol{\beta} = (\beta_1 \ldots \beta_n)$ satisfy the inequality $\boldsymbol{\alpha} \prec \boldsymbol{\beta}$ if and only if there exists some index i such that:

- for the first $i-1$ weight vectors $\mathbf{w}^{(1)}, \ldots, \mathbf{w}^{(i-1)}$, the weighted sum of the components of $\boldsymbol{\alpha}$ is equal to that of the components of $\boldsymbol{\beta}$:

$$\sum_{k=1}^{n} w_k^{(1)} \alpha_k = \sum_{k=1}^{n} w_k^{(1)} \beta_k$$
$$\vdots$$
$$\sum_{k=1}^{n} w_k^{(i-1)} \alpha_k = \sum_{k=1}^{n} w_k^{(i-1)} \beta_k$$

- for the weight vector $\mathbf{w}^{(i)}$, the weighted sum of the components of $\boldsymbol{\alpha}$ is strictly smaller than the weighted sum of the components of $\boldsymbol{\beta}$:

$$\sum_{k=1}^{n} w_k^{(i)} \alpha_k < \sum_{k=1}^{n} w_k^{(i)} \beta_k$$

ANSWERING THE QUESTION

One simplest non-trivial order $\prec$ corresponds to:

- the smallest number $d=1$ of weight vectors $\mathbf{w} = \mathbf{w}^{(d=1)}$
- with an arbitrary number of non-zero weights

The constraint-based grammar $G_{\prec}$ corresponding to $\prec$ is the **HG grammar** corresponding to the weight vector $\mathbf{w}$

The other simplest non-trivial order $\prec$ corresponds to:

- the maximum number $d=n$ of weight vectors
- with a single non-zero weight each

The constraint-based grammar $G_{\prec}$ corresponding to $\prec$ is the **OT grammar** corresponding to the constraint ranking $C_{k_1} \gg C_{k_2} \gg \cdots \gg C_{k_n}$, where k_i is the index of the unique non-zero component of the weight vector $\mathbf{w}^{(i)}$